REPORT DOCUMENTATION PAGE

AFRL-SR-BL-TR-00-

0652

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	20 March 1999	**Final**, 1 May 1997 - 31 December 1998

4. TITLE AND SUBTITLE

NUMERICAL METHODS IN AERO-OPTICS

5. FUNDING NUMBERS

Grant: F49620-97-1-0139

6. AUTHOR(S)

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8. PERFORMING ORGANIZATION REPORT NUMBER

Final: F49620-97-1-0139

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Office of Scientific Research
NM
801 North Randolph Street, Room 732
Arlington, VA 22203-1977

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

20001205 082

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Distribution Unlimited	

13. ABSTRACT ***(Maximum 200 Words)***

This research project was concerned with the development of rigorous mathematical models, computational algorithms and high performance computer implementations for aero-optics imaging applications. The objectives of this project were to conduct research on inverse problems arising in the areas of: 1) adaptive optics deformable mirror control, 2) image post processing using blind deconvolution techniques and phase retrieval, as well as 3) accurate corrections to phase aberration problems encountered in radar systems. The work has applications in defense, including the airborne laser weapons program (ABL), space surveillance, and in civilian technology, including astronomical, industrial, and medical imaging. The project resulted in a variety of new technologies in the form of robust and efficient algorithms, as well as their implementations. Parallelizations of the computational algorithms were investigated and implemented on the IBM SP2 supercomputer at the Air Force Maui High Performance Computing Center. These codes were extensively tested on real optical imaging data made available by Air Force researchers. Packaging the results of our research into reliable software further facilitates the timely and effective transfer of vital new knowledge to DoD research laboratories and to industry.

There were considerable collaborative research and transition activities at Air Force Laboratories during the course of this twenty month project. Several visits by principal investigator Plemmons were made to the Air Force Research Laboratory at Kirtland AFB, NM, for the purpose of close interaction with Air Force researchers on our grant projects.

14. SUBJECT TERMS	15. NUMBER OF PAGES
aero-optics, computational mathematics, image and signal processing, parallel algorithms	30
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UNLIMITED

FINAL REPORT

1 May 1997 to 31 December 1998 (20 months)

Numerical Methods in Aero-Optics

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Numerical Methods in Aero-Optics

A Introduction

This is the final report on the 20 month project "Numerical Methods in Aero-Optics". The termination of funding for this project concludes 16 years of research under sponsorship of the Air Force Office of Scientific Research. A proposal for continuation of the work is pending with the U.S. Army Research Office. However, the principal investigator would also like to have the AFOSR consider continuation of our funding, relative to our record of interaction with Air Force Research Laboratories.

Applied as an image of an object is formed, *adaptive optics* techniques compensate for degradations added along the path of the light from the object being imaged. *Image restoration* (post-processing))tools are then used to scrub the captured optical image even cleaner. The first phase is a massive control problem. The second is a delicate inverse problem. Both adaptive optics and image restoration demand *sophisticated mathematics and state-of-the-art computation.* Figure 1 gives an overall diagram of a typical adaptive-optics closed-loop control system.

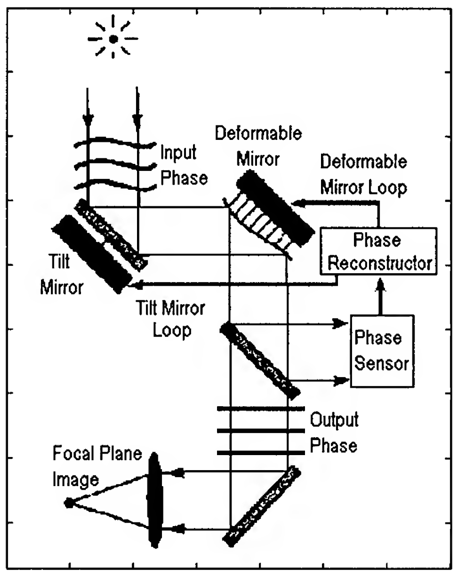

Figure 1: Typical Adaptive Optics System

Image restoration involves the removal or minimization of degradation (blur, clutter, noise, etc.) in an image using a priori knowledge about the degradation phenomena. Blind restoration is the process of estimating both the true image and the blurring operator from the degraded image characteristics, using only partial information about degradation sources and the imaging system. Our main interest concerned optical image enhancement, where the degradation involves a convolution process. Image restoration techniques are also providing clearer views of objects.

The power of these tools is substantial. One of our simulations, for example, shows them improving the resolution of a telescope from being barely able to spot an object the size of a house trailer in earth's orbit to detecting a hand waving from the trailer's window!

Adaptive optics compensation plays an essential role in current state-of-the-art atmospheric telescope imaging technology. The ideal earth-based astronomical telescope is built on bedrock, high on a remote mountain. The solid foundation partially stabilizes the telescope against wind and other potential causes of vibration, while the altitude and isolation minimize atmospheric degradation. The Hubble space telescope carries this logic to its natural extreme, but even the Hubble's accuracy is limited by the effects of thermal stresses and other forces that shift the phase of the incoming light ever so slightly.

Ideally, light from a distant object high above the earth's atmosphere arrives at a telescope's mirror as a single planar wavefront. The only limit on resolution should be diffraction by the telescope mirror aperture. In imaging through the atmosphere, tiny local variations in the index of refraction of the atmosphere induce small phase errors that make the incoming plane wave look more like a sheet of crumpled paper. The mirror then adds phase errors of its own; even a theoretically perfect mirror will be distorted by thermal stresses, not to mention the effects of small vibrations in the telescope structure.

In this setting, active and adaptive optics attempt to compensate for these phase errors using as a reference the phase error in the image of a guide star, either a bright natural star very near the target image or a "star" created by directing a laser into the atmosphere. Guide stars are especially effective against the degradation by atmospheric turbulence of images collected by ground-based telescopes because they provide an estimate of the unknown blurring operator. Furthermore, thermal distortion and gravity can induce small deformations in lightweight mirrors. Active optics corrects these very low frequency errors by delicately nudging the primary mirror with hydraulic actuators.

Adaptive optics corrects the higher frequency errors caused by atmospheric irregularities and telescope vibration. The distortion measured using the guide star drives a control system that adjusts a separate set of mirrors. (The primary mirror is too big to respond fast enough.) With adaptive optics, instruments like the 3.5-m telescope at the Starfire Optical Range of the U.S. Air Force Research Laboratory in New Mexico, can partially correct the image before it is recorded. Note that this real-time control requires extraordinarily high-speed computation – up to 10 billion floating point operations per second.

Further, postprocessing restores the adaptive optics recorded image to a state even closer to perfection by filtering out any remaining noise and blur that can be distinguished from the image. The classic tool is regularized least squares; one of the newest techniques we investigated is based on the solution of a nonlinear partial differential equation. Like adaptive optics, both demand cutting-edge computation.

In the following sections information is provided concerning our activities under this grant on these topics over the past 20 months. We believe that this work in computational mathematics has resulted in several contributions to the state-of-the-art in investigations into algorithms and software for high-performance, high-resolution imaging applications. Our work was particularly directed toward the development and implementation of innovative algorithms and techniques

for optical imaging. The following research topics (each involving the formulation and numerical solution of difficult ill-posed inverse problems) are described in the report:

1. **Adaptive Optics (AO):** Atmospheric turbulence has been a limiting factor for imaging since telescopes were invented. Both atmospheric and telescope induced aberrations distort the spherical wavefront of arriving light. Adaptive optics is a scientific and engineering discipline whereby a distorted optical wavefront is compensated to correct for errors induced by the environment, e.g., a turbulent atmosphere, through which it passes. Our work in this area, joint with Brent Ellerbroek (DoD Starfire Optical Range), Paul Pauca and Xiaobai Sun (Duke), and Moody Chu (NC State), has made contributions to deformable mirror control algorithms, performance analysis, software, and DSP hardware design of on-line adaptive-optics systems. Additional applications include methods for focusing and propagating laser beams.

2. **Image Restoration (IR):** We have tried to make novel contributions to the image postprocessing steps of restoration, enhancement and analysis of images taken through a turbulent medium such as the atmosphere. This work, joint with GRA Paul Pauca (Duke), Tony Chan (UCLA) and Curt Vogel (MSU), has applications in defense, including remote sensing, target identification and laser weapons programs, and to civilian technology, including astronomical, industrial, and medical imaging. We are concerned with fast nonlinear optimization methods for blind deconvolution. We considered novel algorithms for single and multiple imaging channels, based on phase and sensor diversity.

Our purpose was to investigate the areas of adaptive closed-loop deformable mirror control systems, image postprocessing using blind deconvolution techniques and phase retrieval, and accurate corrections to the phase aberration problems encountered in radar systems. Parallelizations of the computational algorithms were developed and implemented on advanced architectures in conjunction with the DoD High Performance Computing Modernization Program. A description of some of our recent work, including color images, graphics and multi-media animations, is on our Web page at:

http://www.wfu.edu/~plemmons/

under Research Projects.

B Objectives

The *objectives* of this project were to conduct rigorous mathematical research on inverse problems arising in the areas of: (1) adaptive optics deformable mirror closed-loop control systems, (2) image postprocessing using blind deconvolution techniques and phase retrieval, as well as (3) accurate corrections to the phase aberration problems encountered in radar systems. High-resolution images are essential in many important applications in defense, science, engineering, law enforcement, and medicine. The need to extract meaningful information from degraded images is especially vital for such DoD applications as aero-optics imaging, surveillance photography, and modern synthetic aperture imaging systems. Sources of image degradation vary among application areas, but include atmospheric turbulence, turbidity in a fluid medium, motion blur, insufficient sampling, electronic noise, and numerous other effects.

The goals of this project are summarized as follows:

- Develop and evaluate innovative adaptive optics deformable mirror control algorithms, with concentration on performance analysis, software, and DSP hardware design of on-line adaptive-optics systems for imaging through turbulence.
- Develop new iterative algorithms with effective regularization strategies for blind deconvolution optical image restoration and enhancement using adaptive filtering and multichannel phase diversity methods.
- Exploit the general mathematical structure of computational problems found in several optical imaging and other signal processing problems to develop a family of methods and routines that are adaptable to several scenarios in obtaining high-resolution images.
- Develop a high performance software package, entitled "Parallel Image Processing Environment (PIPE)", which utilizes the optical image processing algorithms developed in the course of this research.

As we move toward the new millennium, the research results produced under this grant may very well have important impacts on science and engineering as part of a continuing development of the computational foundations of aero-optics technology. Some promising results and new ideas have been put forward and they indicate considerable potential for further progress in solving these important imaging problems in an efficient and stable way on modern computer architectures. The project resulted in a variety of new technologies in the form of robust and efficient algorithms as well as their implementation. Parallelizations of the computational algorithms were investigated and implemented on advanced architectures in conjunction with the DoD High Performance Computing Modernization Program. These were extensively tested on applications from optical imaging. Packaging the results of our research into reliable software will further facilitate the effective and timely transfer of vital new knowledge to DoD research laboratories and to industry.

C Major Accomplishments and New Findings

In the following sections information is provided to help the AFOSR determine whether the results of this DoD research project are consistent with U.S. Air Force missions. We believe that the work described here in computational mathematics has resulted in several contributions to the state-of-the-art in investigations into algorithms and software for high-performance, high-resolution imaging applications. Our work is particularly directed toward the development and implementation of innovative algorithms and techniques for optical imaging, including adaptive optics and image postprocessing.

C.1 Adaptive Optics Imaging Systems

This research is concerned with several related projects in optical imaging. The projects to be described in this section concern adaptive optics control studies in collaboration with Dr. Brent Ellerbroek at the DoD Phillips Laboratory Starfire Optical Range. Much of the work is summarized in our papers [8, 12, 38, 39].

We envision that related applications of our work can be identified at, for example, the DoD Intelligent Optics Laboratory in Adelphi, MD. One important project concerns the efficient computation of covariance matrices that arise in the modeling of atmospheric parameters in adaptive optics (AO) systems. In a more general framework, our work concerns the development and implementation of fast and accurate algorithms for integral discrete transforms with applications in adaptive optics control systems, see [39]. We are currently investigating the potential of our techniques for use, for example, in adaptive optics deformable mirror control. A formal mathematical framework which unifies much of this work is given in our paper [8].

Many modern astronomical telescopes are now built with deformable mirrors that can be adjusted dynamically. Real-time control of the separate actuators of such a mirror can accommodate distinctly different sources of error, such as wind shake and time-varying atmospheric distortion. Each source has its own characteristic temporal frequency; those of wind shake, for example, are typically much higher than those of atmospheric turbulence. The key to adjusting the mirror actuators on the fly is to choose a basis set of mirror deformations, known as mirror control modes, which best control each disturbance at a bandwidth matched to its characteristic frequency. In contrast, correcting all disturbances at a common bandwidth will either allow high frequency errors to sneak through if this bandwidth is set too low, or add unnecessary noise to the correction of low frequency errors if the bandwidth is too high.

Brent Ellerbroek of the Starfire Optical Range, principal investigator Plemmons, and other co-authors, have developed a multiple bandwidth modal control strategy that can minimize mean squared phase error in the image across multiple sources of error [12, 41]. These techniques advance previous approaches by enabling the simultaneous optimization of both the mirror control modes and the associated control bandwidths without choosing in advance the basis set of control modes.

The optical performance of the system at a particular bandwidth is characterized by a matrix. The optimal control modes could be determined by finding the one unitary transformation that comes closest to simultaneously diagonalizing all of these optical performance matrices. However,

the approximate simultaneous diagonalization of more than two matrices is not an easy task to formulate algorithmically, and any such scheme would be computationally expensive. Instead, we use a novel trace maximization approach based on a hill climbing scheme relative to pairs of matrices in order to minimize mean squared phase error.

In particular, we have studied in publications [12, 41] a non–smooth optimization problem arising in adaptive optics, which involves the optimal real–time control of deformable mirrors designed to compensate for atmospheric turbulence and other image degradation factors, such as wind–induced telescope vibration. The surface shape of this mirror must change rapidly to correct for time–varying optical distortions caused by these sources of image degradation. One formulation of this problem yields a functional $f(U) = \sum_{i=1}^{n} \max_j \{(U^T M_j U)_{ii}\}$ to be maximized over orthogonal matrices U, where U and a fixed collection of $n \times n$ symmetric matrices M_j. We consider the situation which can arise in practical applications where the matrices M_j are "nearly" pairwise commutative. Besides giving useful bounds, results for this case lead to a simple corollary providing a theoretical closed–form solution for globally maximizing f if the M_j are simultaneously diagonalizable. However, even here conventional optimization methods for maximizing f are not practical in this real–time environment. The general optimization problem is quite difficult and is approached using a heuristic Jacobi–like algorithm. Numerical tests using the algorithm indicated that the performance of adaptive optics systems, such as those of interest to the Air Force, can be improved by the use of our Jacobi–like algorithm.

These schemes can show substantial improvement over single bandwidth control methods. They also lend themselves to parallel implementation and real-time computing, see [12, 41].

C.1.1 Adaptive Deformable Mirror Control

Specially designed deformable mirrors operating in a closed-loop adaptive optics system can partially compensate for the effects of atmospheric turbulence. The systems detect the distortions using either a natural guide star (point) image or a guide star artificially generated from the back scatter of a laser generated beacon. To illustrate the basic idea, we sketch the main components in an AO system are in Figure 2. These include the *deformable mirror* (DM), the *wave front sensor* (WFS), and the *actuator command computer.* Light in a narrow spectral band passing through the atmosphere is modeled by a plane wave. When traveling through the atmosphere that does not have a uniform index of refraction, light waves are aberrated and no longer planar. In a closed-loop AO system, this aberrated light is first reflected from the DM. Some of this light is focused to form an image, and some is diverted to the WFS that measures the wave front phase deformation. The actuator command computer takes measurements from the WFS and map them into real time control commands for the DM. How this translation is done depends on the criterion selected.

In general, wave front sensing is a key aspect of many optical techniques and systems such as optical shop testing, interferometry and imaging through random media such as the earth's atmosphere. We have used the massively parallel IBM SP2 at the DoD's Maui High Performance Computing Center (MHPCC) for simulation studies in order to better understand the characteristics of our algorithms. In a paper [12] which appeared in the J. Optical Soc. Amer. A, we have

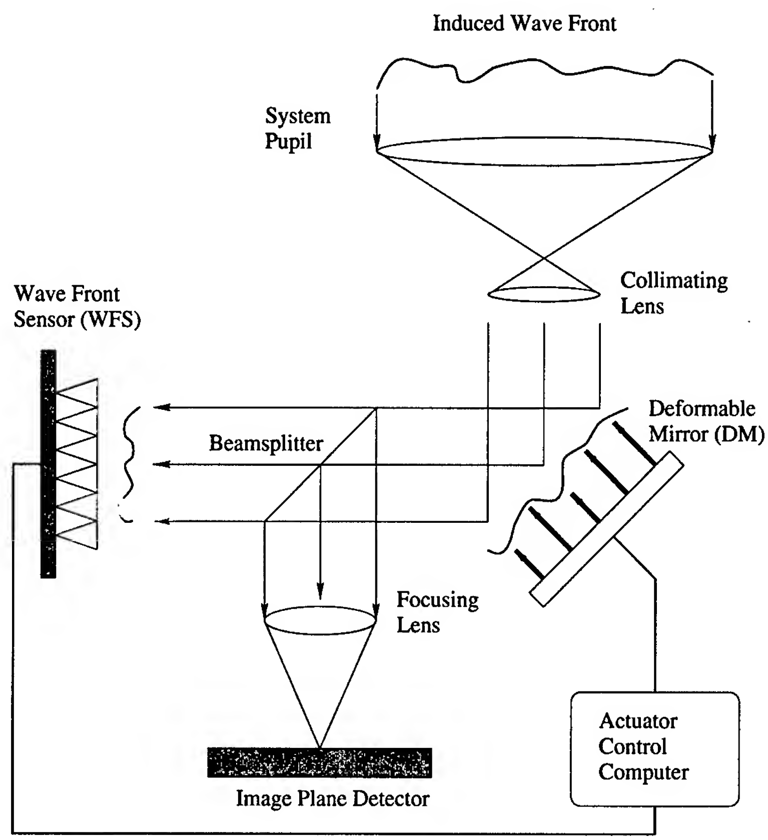

Figure 2: A simplified closed-loop AO system with main components.

helped to develop a theory with applications to closed-loop adaptive control methods to adjust the shape of the telescope deformable mirrors in real-time. A second paper [41] was presented at the SPIE Annual Meeting in San Diego, July 1998. Numerical tests using this algorithm indicate that in the presence of windshake jitter, the performance of a closed-loop adaptive optics system can be improved by the selection of distinct and independently optimized control bandwidths for separate modes of the wave front distortion profile. Related interaction on minimal variance optimal wavefront reconstructor updating methods was discussed at February 1998 and March 1999 visits to the DoD Starfire Optical Range in New Mexico. Work with potential applications to optimal reconstructor approximation methods was the objective of these visits.

C.1.2 Modeling and Adaptive Optics System Design

AO has emerged as the technique of choice to mitigate the blurring caused by atmospheric turbulence in large aperture imaging systems that allow extremely dim objects to be observed [23]. Modeling and evaluating the expected performance of AO systems is essential to the proper design of AO systems for large telescopes such as Gemini, Keck, Subaru, and VLT [11]. The objective of AO system design is to select hardware parameters and control approaches that will optimize

system performance for the expected operating conditions subject to the resources available, and maximize the resulting payoff for the intended scientific applications. The large range of possible observing scenarios and AO system parameters entails numerous cases to be considered, and fast computational approaches for performance evaluation and optimization are highly desirable.

Modeling AO system performance is an essential part of AO system design. Among other modeling approaches, the linear systems model framework developed by Ellerbroek [13] provides first-order performance estimates which account for the full range of fundamental adaptive optics error sources and their interacting effects. However, such system performance evaluation and optimization models require intensive computations based on the covariance matrices for the statistical relationship between the WFS gradient measurements which drive the AO control loop and the turbulence-induced phase distortions to be corrected. Ellerbroek has given a general element-wise formulation for each ijth entry of the covariance matrices in the form

$$\begin{aligned} \alpha_{ij} &= c_1 \left[\int_0^\infty C_n^2(h)dh\right]^{-1} \int\int w_i(\mathbf{x}) w_j(\mathbf{x}') \int_0^H C_n^2(h) \\ &\times \int_0^\infty f(s) \left[J_0\left(\frac{2\pi y}{L_0}s\right) J_0\left(\frac{2\pi y'}{L_0}s\right) - 1\right] ds\, dh\, d\mathbf{x}\, d\mathbf{x}', \end{aligned} \tag{1}$$

where J_0 is the zero-order Bessel function of the first kind, and the quantities y and y' are scalar functions of $\mathbf{x}, \mathbf{x}' \in R^2$, and $h \in R$. See Ellerbroek[14] for notation and further details. Denote the *two-parameter Hankel transform* [10] of a function $f(x)$ by

$$h(a,b) = \int_0^\infty f(s) J_0(as) J_0(bs) ds, \tag{2}$$

where a, b and s are variable nonnegative real numbers. For the inner integral with respect to s in (1),

$$a = \frac{2\pi y}{L_0}, \quad b = \frac{2\pi y'}{L_0}. \tag{3}$$

The two-parameter Hankel transform is invoked for each and every covariance matrix entry in the linear system model of Ellerbroek[13, 14]. Due to the fact that y and y' are functions of $\mathbf{x}$, $\mathbf{x}'$, and h, the efficient and accurate evaluation of the two-parameter Hankel transform becomes critical to the evaluation of the outer integrals in (1). For covariance computations, it is desired to efficiently evaluate (2) in a block-wise manner for N values of a and b, where N is proportional to a subset of deformable mirror actuators and wavefront sensor measurements, thus obtaining an $N \times N$ matrix

$$H(\mathbf{a}, \mathbf{b}), \quad \mathbf{a}, \mathbf{b} \in R^N. \tag{4}$$

Products involving the matrix $H(\mathbf{a}, \mathbf{b})$ are core operations for the integral computation in (1) after appropriate discretization. Figure 3 illustrates a particular example of the matrix $H(\mathbf{a}, \mathbf{b})$ that arises in adaptive optics simulations [13, 48].

The computation of the covariance matrices is intensive in two aspects. First, the computation for each given parameter set invokes the task of generating each and every entry of the covariance matrix, which is followed by calculations using the generated matrix for performance evaluation.

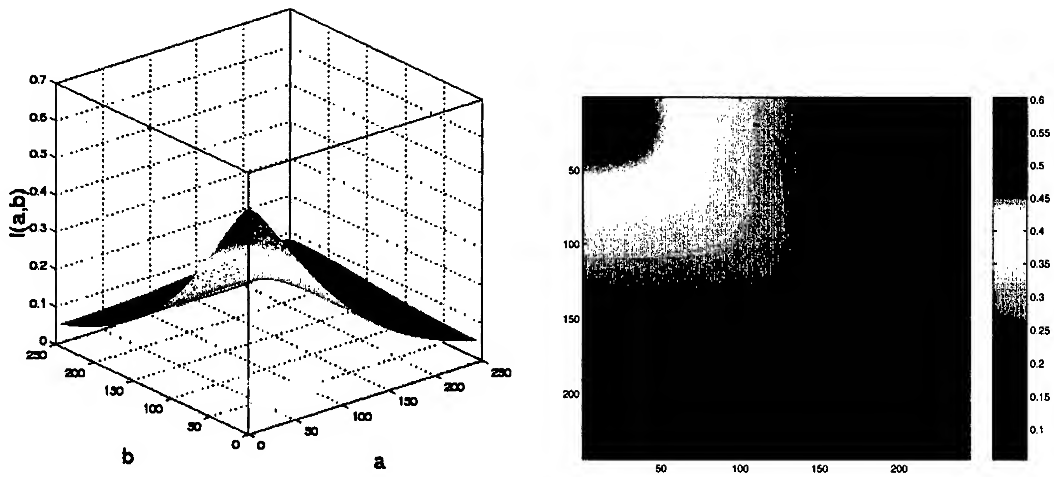

Figure 3: The two-parameter Hankel transform matrix $H(\mathbf{a}, \mathbf{b})$.

Secondly, such computation is carried out many times over a large sampling space of AO performance parameters. To get a glimpse of the parameter space, we list some of the parameters in groups according to the physical meaning: (1) observing scenario parameters such as wavelength, aperture diameter, and zenith angle; (2) assumptions on the atmosphere such as the wind profile, the $C_n^2(h)$ (or turbulence) profile, and the turbulence outer scale; (3) architecture components of wave front sensing such as the WFS subaperture geometry, beacon geometry, pupil conjugate range and noise level; (4) architecture components of deformable mirrors such as the actuator geometry and conjugate range, and (5) level of hardware imperfection or limitations.

In terms of the computation complexity, the size of the covariance matrices is proportional to the number of deformable mirror actuators and wavefront sensor measurements. Typically, the matrices are of order from 100 to 5000 for high-order AO systems designed to compensate for turbulence on very large telescopes. Each matrix entry invokes the evaluation of multiple integrals as seen in (1). Present computational approaches to evaluating each matrix element may sample the integrand at up to 10^4 points, where the integrand itself may be computed as a single or double infinite sum [14]. The subsequent computations with the covariance matrices require numerous inversions and multiplications of large matrices [13] to obtain performance estimates such as the residual mean-square phase error and the associated optical transfer function (OTF) of the telescope. To cover a parameter sampling space of a reasonable size, there may be hundreds or thousands of such covariance matrices to generate and compute with using today's computational practice for performance evaluation.

We have proposed novel approaches for efficient computations that exploit (i) the mathematical relationship among the entries of the matrix for each parameter set and (ii) the common computation procedures and intermediate results among different sets of parameters. In preliminary work in [38], we proposed the use of fast Hankel transform methods [26, 27] combined with a matrix representation of $H(\mathbf{a}, \mathbf{b})$ in factored form, leading to fast computation of covariance matrices of

the form (1). Such matrix factorization formulation aids not only in revealing complexity and approximation accuracy (such as errors introduced by truncation and discretization), but also in identifying operations for which fast algorithms are known. A low rank matrix factorization may allow for efficient computation of matrix-vector products involving $H(\mathbf{a}, \mathbf{b})$ in the computation of covariances. Furthermore, its compact form representation also allows for storage savings.

We derived in [38] the following factored form of $H(\mathbf{a}, \mathbf{b})$:

$$H(\mathbf{a}, \mathbf{b}) = H_T(\mathbf{a}, \mathbf{b}) + E, \tag{5}$$

where

$$H_T(\mathbf{a}, \mathbf{b}) \quad = \quad M D_u C^{\mathrm{T}} D_N C D_v M^T. \tag{6}$$

The factors D_u and D_v are diagonal matrices, and C is the discrete cosine transform matrix. The middle factor D_N is also diagonal and holds the tabulated values of $f(s)$. The factor M is the one-dimensional equivalent of the matrix involved in particle simulations [20],

$$M_{ij} = \begin{cases} \dfrac{1}{i^2 - j^2}, & j < i \\ 0, & \text{otherwise} \end{cases} . \tag{7}$$

Note that the term E corresponds to a k-th order end-point quadrature correction and takes only $O(N)$ to apply and store. A significant advantage of our approach is that the factorization in (5) is given in terms of *fast* operators, all of which can be applied to a vector efficiently in at most $O(N \log N)$ operations. Hence, computing a matrix-vector product involving $\mathbf{H}(\mathbf{a}, \mathbf{b})$ can also be done in $O(N \log N)$ operations.

A related fast approach for subsequent computations involving $H(\mathbf{a}, \mathbf{b})$ is based on a power series expansion [14]. In matrix form, the expansions can be done at different levels of granularity. We illustrate our partition template for the expansion in Figure 4. In other words, one can partition the matrix into smaller *low rank subblocks* which can be stored and applied efficiently. Using the ideas behind the *fast multipole algorithm*, the matrix can be partitioned into blocks whose size doubles as the blocks get further away from the diagonal blocks. Thus, the total cost for matrix-vector multiplication can be reduced to $O(N \log N)$ using the hierarchical partitioning and rank-revealing scheme just described.

We will continue to extend the ideas presented here with the aim of producing robust and modular software for efficiently forming and applying covariances matrices required in AO system design. Our preliminary studies by Pauca, Ellerbroek, Plemmons and Sun in [38, 39] have shown that novel fast Hankel transform techniques can *reduce the computational complexity* of currently used techniques by Ellerbroek [13, 14] from $O(N^4)$ to $O(N^2 \log N)$.

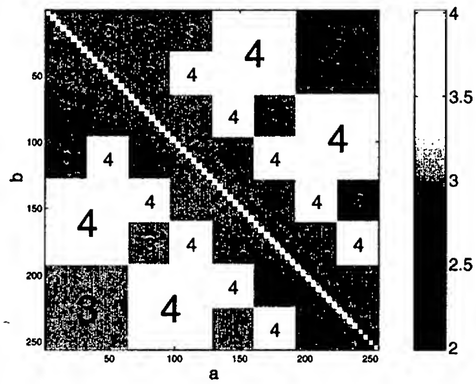

Figure 4: Template of low-rank subblocks of $H(\mathbf{a}, \mathbf{b})$ with f given in [13]. The numbers represent the numerical ranks of the subblocks.

C.2 Image Restoration

This research topic is concerned with the development and implementation of computational methods for the solution of certain large and often ill-posed inverse problems [15, 42] in image reconstruction and restoration, [2, 29], where the solution may not depend continuously on the data. In general, applications abound in science, engineering and medicine, for identifying and measuring useful features from image data, such as military targets, terrain features, handwritten alphanumeric characters, or medical artifacts such as tumors. This work also includes technologies for information management, data fusion, and visualization of extracted information.

The techniques of adaptive optics are helping ground-based sensors to capture better, but hardly perfect, astronomical images. More mundane devices like surveillance cameras are cursed from the start with gritty, low-resolution images. In both settings, image restoration techniques can help obtain a clearer picture by separating the image from the degradations.

Classic restoration techniques often model the received image as the sum of noise and a blurring operator acting on the true image. (The blurring operator is a convolution with the point-spread function that characterizes the aberrations.) These techniques seek to recover the correct image by solving an ill-conditioned least squares problem, usually regularized through the addition of some smoothness requirement to be satisfied by the restored image.

This linear problem is a formidable computation because of the size of the blurring operator – its dimension is the number of pixels in the image – and its inherent ill conditioning. Regularization can render the restoration problem solvable in practice, by neutralizing some of the ill-conditioning, but it also removes sharp edges and similar distinguishing features. Once distinctive characteristics, for example, can become unrecognizable.

Leonid Rudin of Cognitech, Inc., and Stanley Osher of UCLA, formerly of Cognitech, have developed a widely used alternative known as the *total variation* (TV) technique because it minimizes the total variation of the image instead of its second derivative [44, 45]. Rather than insisting on a completely smooth image, TV requires only that it have bounded variation, permitting sharp

edges but eliminating spurious oscillations. They came to their approach in part through methods used for tracking shock fronts in gas dynamics calculations, a setting that also seeks to preserve sharp boundaries without introducing extraneous detail.

The problem at the heart of a TV image restoration problem is equivalent to a nonlinear partial differential equation (the Euler equation of the constrained minimum variation problem). The restored image solves this steady-state problem. In their original work, Rudin and Osher found that steady-state solution by iterating in time from an appropriate initial condition. More recently, various researchers, including Tony Chan of UCLA, Curt Vogel of Montana State University, and principal investigator Plemmons, have proposed preconditioned conjugate gradient methods for iterating to the solution [1, 7, 30, 31, 33, 49, 50].

The down side of TV-based image restoration is its computational expense. For example, Rudin and Osher's time-stepping scheme for solving the TV differential equation can be slowed by stability restrictions that force it to take relatively small time steps.

In this respect, principal investigator Plemmons and his colleagues James Nagy of Southern Methodist University, Paul Pauca of Duke University, and Todd Torgersen of Wake Forest have proposed a compromise: Use TV methods to sharpen the estimate of the blurring operator obtained from an auxiliary source like a guide star, then apply quicker linear restoration algorithms to the image (see [30, 31, 33]).

Since the blurring operator is localized, TV techniques can be applied to it fairly cheaply, thereby gaining from the ability of minimum total variation to preserve sharp transitions without the expense of a complete TV restoration. Using the TV estimate of the blurring operator, linear restoration is then applied adaptively to subregions of the full image allowing subregions of the image to converge at a more natural rate, a technique the authors call the *space-varying regularization* (SVR) technique. This novel SVR method reduces the possibility of excessive smoothing and magnification of noise during the linear restoration phase.

Plemmons and his colleagues solve the linear restoration subproblems iteratively using SVR. The challenge is to continue the iterations long enough to amplify the components associated with the image but not so long that the noise is amplified as well. By simultaneously monitoring the size of the image components and the residual, or equation error, they can choose a stopping criterion appropriate for the portion of the image under study. This approach stops iterations early for a region with little spatial variation, such as an image of empty sky, but lets them run longer for a busier region that includes, say, a piece of the satellite or star under observation.

Simulations using the 3.5-m telescope at the Starfire Optical Range show impressive improvements: Combined with the multiple-bandwidth adaptive optics control of Ellerbroek, Plemmons, and others, use of SVR strengthens resolution by a factor of about 50. A telescope that can discern nothing smaller than 30 meters at a range of 1000 km using its optics alone finds its resolution improved to 20 *centimeters* when a combination of adaptive optics and SVR postprocessing is used.

Many image restoration problems fall into the broad category of blind deconvolution because it is necessary to estimate both the true image as well as the blur from the degraded image using only partial information about the blurring operator. Blind deconvolution approaches include those developed by Julian Christou of Phillips Laboratory and by Plemmons and his colleagues

Michael Ng of the Australian National University and Sanzheng Qiao of McMaster University (see [35, 36]). These particular approaches couple constrained optimization with nonlinear conjugate gradient methods. A major part of the the principal investigator's recent image postprocessing work has been concerned with developing new, effective blind deconvolution methods.

C.2.1 Current Status of our Restoration Work

Image restoration and enhancement is an essential aspect of image analysis [2, 3, 12, 29, 43]. It involves the removal or minimization of degradation (blur, clutter, noise, etc.) in an image using a priori knowledge about the degradation phenomena. Blind restoration is the process of estimating both the true image and the blurring operator from the degraded image characteristics, using only partial information about degradation sources and the imaging system [28]. Our main interest in image postprocessing is in optical image enhancement (restoring the phase profiles of electromagnetic waves passing through a turbulent medium), where the degradation often involves a convolution process e.g., [43]. Our related interests include work on electromagnetic wave propagation, absorption, and scattering.

The image formation process is often modeled as a Fredholm integral equation of the first kind:

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y;s,t) f(s,t) ds dt + \eta(x,y) \tag{8}$$

where $g(x,y)$ is the observed (degraded) image, $f(x,y)$ is the true (original) image (unknown), and $\eta(x,y)$ is assumed to be additive, Gaussian noise. Here, $h(x,y;s,t)$ represents the blurring point spread function (PSF). Our particular application concerns image restoration computations. The interest is on the development of fast algorithms and their extension to iterative blind deconvolution, where the blurring operator h as well as the image f is to be estimated.

If we let $\mathcal{H}$ denote the blurring operator, then the image restoration problem associated with (8) can be expressed as a linear operator equation

$$g = \mathcal{H} f + \eta. \tag{9}$$

Let u and v denote two-dimensional variables. If $\mathcal{H}$ is a convolution operator, as is often the case in optical imaging [12, 43], then the operator acts uniformly (i.e., in a spatially invariant manner) on f. Here, (9) can be written as

$$\mathcal{H} f(u) = \int_{\Omega} h(u-v) f(v) dv \ , \tag{10}$$

where Ω is the domain of f. The problem is to both deconvolve and denoise the recorded image during the reconstruction process, and we refer to this as the *denoising and deblurring* problem. In optical imaging, the kernel h in (10) is called the convolution *point spread function* (PSF). The Fourier transform of h is called the *optical transfer function* (OTF). After discretization of (9), the spatial operator $\mathcal{H}$ defined by h in (10) is a matrix that we denote by H. Here, in the spatially invariant case, H is *block Toeplitz with Toeplitz blocks.* Thus the fast Fourier transform (FFT)

can be used in computations involving H, e.g., [4, 5, 47], with efficient implementation possible on high performance architectures [19].

A classical approach employed for solving (9) is penalized least squares, also called Tikhonov regularization in the inverse problems literature [15]. It requires minimization of an expression

$$\|\mathcal{H}f - g\|^2 + \alpha J(f), \tag{11}$$

where $\|\cdot\|$ denotes the norm on $L^2(\Omega)$, α is a positive (regularization) parameter. The functional $J(f)$ serves the purpose of stabilizing the least squares problem and penalizing certain undesirable artifacts like spurious oscillations in the computed f. Various choices of $J(f)$ can be made, including $\|Sf\|^2$, where S is some smoothing differential operator, or the identity. This model, based on the Euclidean norm, leads to fast linear methods for computing f, and is often the method of choice by practitioners [2, 29]. However the use of other norms, such as the $\mathcal{L}_1$ norm, lead to nonlinear minimization methods which sometimes result in superior enhancement of blocky, noisy images, but with added computational cost. Rudin, Osher and Fatemi [45] have suggested an image enhancement method based on solving a nonlinear PDE constrained minimization problem where the function being minimized is the *Total Variation* (TV) of the image $f = f(x,y)$. Numerous other papers on various TV approaches for image denoising or restoration have been written in the past five years, including, [1, 7, 30, 44, 49, 50]. We have chosen to use the numerical approach developed by Vogel and Oman [49, 50] for solving the nonlinear TV optimization problem. The outer iteration is a fixed point method and the inner iteration involves a preconditioned conjugate gradient method [22], applied to a large scale system

$$(H^*H + \alpha L_\beta(f^k))f^{k+1} = H^*g, \qquad k = 0, 1, ...$$

For deconvolution, H is block Toeplitz with Toeplitz blocks. Here, α and β are chosen to stablize the computations. The matrix $\alpha L_\beta(f^k)$ is a 2-D nonconstant Laplacian with five bands, associated with an elliptic operator. Its spectrum can vary widely over the outer iterations, i.e., with the index k, causing numerical difficulties that must be considered [49, 50].

Our recent work in [30, 31] concerns a new *space-varying regularization* (SVR) approach to image restoration using multiscale methods, and associated techniques for accelerating the convergence of iterative image postprocessing computations. Denoising methods, including TV minimization, followed by segmentation-based preconditioning methods for minimum residual conjugate gradient iterations [22], are investigated in our work. Regularization is accomplished by segmenting the image into (smooth) segments and varying the preconditioners across the segments. The method appears to work especially well on images that are piecewise smooth. The algorithm has computational complexity of only $O(\ell n^2 \log n)$ per iteration, where n^2 is the number of pixels in the image and ℓ is the number of segments used. Also, functional and data parallelization of the algorithm is possible.

Our approach is especially attractive for restoring images with low signal to noise ratios, and magnification of noise is effectively suppressed in the iterations, leading to a numerically efficient and robust regularized iterative restoration algorithm. These ideas are being extended to a novel Krylov subspace approach, to enhance one of our approaches to blind deconvolution. We began

another approach in [35, 36], using a nonlinear inverse filtering technique. We are also studying a phase diversity blind image deconvolution method in [6]. A recent SIAM News article [9] discusses, in part, some of our prior contributions in high-resolution imaging, reported in papers [25, 30, 31, 32, 33, 35, 36]. The additional feature of estimating the PSF as well as deblurring the image in blind deconvolution adds another level of difficulty to the restoration process, leading to some exciting new challenges.

C.2.2 Novel Blind Deconvolution Methods

A fundamental issue in image restoration is blur removal in the presence of observation noise, using only partial information about degradation sources and the imaging system, i.e., the blurring operator is essential unknown. In the important case where the blurring operation is spatially invariant, then the basic restoration computation faces the usual difficulties associated with ill-conditioning in the presence of noise [3]. The image observed from a shift invariant linear blurring process, such as an optical system, is described by how the system blurs a point source of light into a larger image. The image of a point source is the *point spread function*, denoted by h. The observed image g is then the result of convolving the PSF h with the "true" image, say f. This blurring process is represented by the convolution equation $g = h \star f$. The standard deconvolution problem is to recover the image f given the observed image g and the blurring operator h. The PSF of an imaging system can sometimes be described by a mathematical formula. More often, the PSF must be estimated empirically. Empirical estimates of the PSF can sometimes be obtained by imaging a relatively bright, isolated point source. In astro-imaging, the point source might be a natural guide star, or a guide star artificially generated using range-gated laser backscatter, e.g, [12, 43]. Notice here that the PSF as well as the image may be degraded by noise.

Recursive Inverse Filtering In many applications data corresponding to h is not completely known. *Blind deconvolution* is the process of estimating both the true image f and the blur h from the degraded image g. One direction of our recent work on blind deconvolution, reported in [35, 36], is to incorporate regularization into and refine a nonlinear recursive inverse filter blind deconvolution method first proposed by Kundur and Hatzinakos [28]. They call their scheme the *nonnegativity and support constrained, recursive inverse filtering method*, or NAS-RIF, for short. (Also, some important *direct filtering* approaches to iterative blind image deconvolution have recently been proposed in [7, 16, 24, 53]).

An algorithm for regularized iterative blind deconvolution using truncated eigenvalue and total variation regularization in conjunction with inverse filtering is developed in our paper [35]. The scheme is based upon a constrained optimization method, using a minimization procedure with nonlinear conjugate gradients (see [37], Chapter 5). We apply regularization to the inverse filter by using an inexpensive *eigenvalue truncation* scheme, and allow the user the option of applying *total variation regularization* to the estimated image. We call our approach the *nonnegativity and support regularized recursive inverse filter* (NSR-RIF) algorithm. The constrained optimization problem associated with our algorithm is *convex*; thus, a variety of nonlinear optimization schemes can be used to minimize the objective function. See Table 1 for an outline of the proposed algorithm

and Figure 5 for an schematic diagram. We remark that the regularized image estimate can replace $\tilde{y}$ as an input to compute the error vector z in the recursive inverse filter algorithm. The switch in Figure 5 indicates this option.

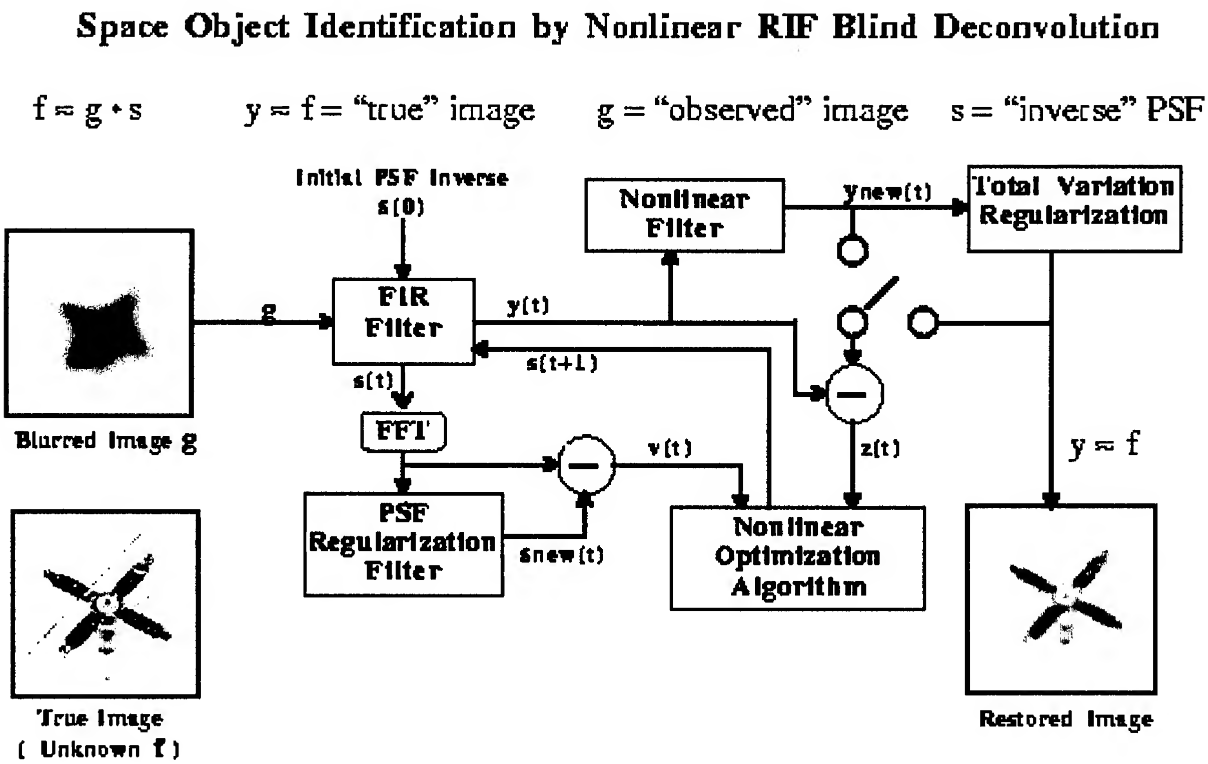

Figure 5: Diagram of the NSR-RIF algorithm.

Numerical tests are reported in [35, 36] on some simulated and optical imaging problems, and a comparison is made with the NAS-RIF algorithm [28]. Figure 6 also gives a sample restoration of a ground-based image of a satellite using our NSR-RIF blind deconvolution method from [35]. In this example a 256×256 image is considered. Specifically, the true object is an ocean reconnaissance satellite. A computer simulation algorithm was used to produce an image of the satellite, shown in Figure 6(left), as it would be observed from a ground-based telescope using adaptive-optics compensation. The satellite was modeled as being 12 meters in length and in an orbit 500 kilometers above the surface of the earth. The charge-coupled device (CCD) for forming the image was a 65,536 pixel square array. CCD root-mean-square read-out noise variance was fixed at 15 microns per pixel to reflect a realistic state-of-the-art detector. The computed restoration is shown in Figure 6(right).

Recently there has been growing interest and progress in using *total least squares* (TLS) meth-

Definitions:

g: the blurred and noisy signal of size k.
n: the support size.
p: the filter length is $2p+1$.
$s(t)$: the FIR filter parameter vector of dimension $2p+1$ at the t-th iteration.
$S(t)$: the corresponding matrix of the FIR filter $s(t)$.
$\tilde{y}(t)$: the estimate of the original signal at the t-th iteration.
$J_{reg}(t)$: the objective function at $s(t)$.
$\bigtriangledown J_{reg}(t)$: the gradient vector of $J_{reg}(t)$.
ϵ: the tolerance for the termination.

Initial Conditions:

Set $s(0)$ to all zeros with a unit spike in the middle,
or, to an estimate corresponding to the inverse of the PSF.

Iterations:

For $t = 0, 1, ...$
Compute $y(t) = S(t)g$.
Project $y(t)$ onto $\tilde{y}(t)$ for nonnegativity.
Compute the error $z(t) = y(t) - \tilde{y}(t)$.
Compute $\hat{s}(t) = Fs(t)$, where F is the DFT operator.
Project $\hat{s}(t)$ onto $\tilde{\hat{s}}(t)$ for finite support and TV regularization.
Compute the error $v(t) = \hat{s}(t) - \tilde{\hat{s}}(t)$.
If $J_{reg}(t) < \epsilon$, **then** stop; **otherwise** compute $\bigtriangledown J_{reg}(t)$.
If $t = 0$, **then** set the conjugate gradient direction vector $d(t) = -\bigtriangledown J_{reg}(t)$;
otherwise compute $e(t) = [\bigtriangledown J_{reg}(t) - \bigtriangledown J_{reg}(t-1)]^* \bigtriangledown J_{reg}(t) / \| \bigtriangledown J_{reg}(t) \|_2$;
and set the direction vector $d(t) = -\bigtriangledown J_{reg}(t) + e(t)d(t-1)$.
Perform a line minimization to determine δ_t such that

$$J_{reg}(s(t) + \delta_t d(t)) \leq J_{reg}(s(t) + \delta d(t)), \quad \forall \delta \in \mathcal{R}.$$

Compute $s(t+1) = s(t) + \delta_t d(t)$.
End For

Table 1: **Proposed NSR-RIF Inverse Filter Algorithm for Blind Deconvolution.**

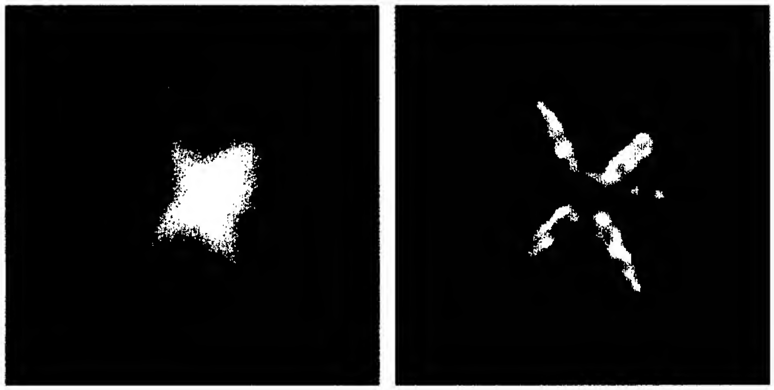

Figure 6: Degraded Image (left) and Restored Image (right).

ods for solving blind deconvolution problems arising in image restoration. Here, the true image is to be estimated using only partial information about the blurring operator, or point spread function, which is subject to error and noise. In this respect, we have contributed a new iterative, regularized, and constrained TLS image restoration algorithm. Neumann boundary conditions are used to reduce the boundary artifacts that normally occur in restoration processes. Numerical tests are reported on some simulated optical imaging problems in order to illustrate the effectiveness of the approach, as well as the fast convergence of our iterative scheme, as indicated in our recent paper [34]. Preliminary numerical results indicate the effectiveness of the method. Future work on this project will exploit the constrained total least squares technique in phase diversity-based deconvolution arising from astronomy, extending work by Chan, Plemmons, and Vogel [6].

Our next steps are also to extend the phase diversity, total least squares and nonnegativity and support regularized recursive inverse filter blind deconvolution methods in [6, 34, 35, 36] to allow for *multiple frames of data*, where a large number of short exposure images are obtained over a short time period, as in video imaging [2]. This work overlaps with our research topic discussed next - multiframe (multichannel) blind deconvolution.

Multichannel Blind Deconvolution and Phase Diversity Another approach to blind deconvolution that we have pursued capitalizes on temporal or *sensor diversity* to obtain stable reconstructions with enhanced resolution. It is based on a multichannel formulation, where noisy, differently blurred versions of the same image are available. In particular, sensor diversity arises in remote sensing, where the same scene is observed at successive time instants through the turbulent atmosphere, with a different transfer function at each instant (this can include the effect of partial compensation by adaptive optics [12, 43]). In some cases, the transfer functions differ primarily by their phase, giving rise to so called *phase diversity.* Phase diversity involves the simultaneous collection of two (or more) short-exposure images. One of these images is the conventional image that has been blurred by unknown aberrations. An image is collected in a *separate channel,* by blurring the first image by a known amount, e.g., using a beam splitter and an out-of-focus lens, which is a quadratic blur. It is somewhat remarkable that estimates for the object *and* the un-

known aberrations can be made from these two images. This is a research topic of considerable current interest in the imaging community, see [6, 9, 40, 46].

Using the two images collected by the phase diversity method as data, one can set up a mathematical optimization problem (typically using a maximum likelihood formulation) for recovering the original image as well as the phase aberration. In the seminal work reported by the authors of [40, 46] an *expectation-maximization* algorithm is constructed to recover both the aberrations (blur) and the fine-resolution image common to both images.

The phase diversity procedure is briefly described as follows [40, 43, 46]. The incoherent isoplantic image formation process is well modeled by the following convolution (see [43]):

$$g_{jk}(u) = \mathcal{H}_{jk}(u-v)f(v) + \eta_{jk}, \tag{12}$$

where f is the object array, $\mathcal{H}_{jk}$ is an incoherent point spread function, corresponding to the jth atmospheric realization and the kth diversity channel, g_{jk} is the corresponding observed image, and u and v are 2-dimensional coordinates. The data set should be chosen to contain sufficient image frames (data) from a total of J atmospheric realizations. But, typically, only $K = 2$ diversity channels are needed. Phase diversity is introduced in the system's *coherent transfer function* (see [40]),

$$P_{jk}(z) = P(z)e^{\{i[\phi_j(z)+\theta_k(z)]\}}, \tag{13}$$

where $P(z)$ is a binary function that serves as a model of the telescope pupil function, $\phi_j(z)$ is the unknown phase-aberration (blurring) function associated with the jth atmospheric realization, $\theta_k(z)$ is a *known* phase diversity function associated with the kth diversity channel, and z is the discrete spatial-frequency variable. The incoherent point spread function $\mathcal{H}_{jk}$ in (12) is just the *squared modulus of the Fourier transform* of the coherent transfer function given by (13). The objective is then to use (12) and (13) to set up an optimization problem for recovering the original image f as well as the phase aberration function ϕ.

The expectation-maximization algorithm is often used to recover both the aberrations ϕ_j and the fine-resolution image f common to the images [46]. This is a computationally intensive algorithm. Our beginning work in [6] aims at improving the numerical efficiency of the restoration procedure. We have begun a project to develop fast computational algorithms for this phase diversity-based blind image deconvolution approach. In particular we addressed the following problems:

- Regularization of the phase function in (13). This improves the stability of the mathematical optimization problem and eliminates some of the spurious minima.
- Quasi-Newton methods, including Gauss-Newton, BFGS and nonlinear EN-like methods [51, 52], that take advantage of the special structure of the optimization problem. These combine fast convergence with relatively low cost per iteration.
- Effective parallel implementation of multilevel preconditioners to quickly solve the large structured linear systems which arise at each quasi-Newton iteration.

- A theoretical and parallel performance analysis of the interaction of particular iterative methods for nonlinear and linear system solving, with the preconditioner and architecture e.g., combinations like nonlinear EN-like methods, linear EN-like methods, and various preconditioners [51].

Multichannel blind deconvolution techniques present both computational and theoretical challenges. They can require solution of large least-squares problems for structured matrices that are block Toeplitz with Toeplitz blocks. Because of the huge sizes of some of the matrices arising in the problems (e.g., $kn^2 \times n^2$ for k $n \times n$ frames of images, where 1024×1024 images are not uncommon), careful implementations of iterative algorithms that avoid forming these matrices explicitly, and which use preconditioning for speedup, are required. We developed methods incorporating statistical considerations to improve robustness to noise. Likewise, in the future, we expect to develop methods for model order reduction, to both improve algorithm stability, and to offer a tradeoff between performance and computation (see [17, 18, 21]). We have also explored further the incorporation of nonlinear (and sparsity-based) regularization techniques [30, 31, 32, 33]. These steps provide yet additional ways to further enhance our methods for obtaining high-resolution aero-optics images.

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F Publications

Most of the papers listed below can be found on our Web page at:

http://www.wfu.edu/~plemmons

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G Presentations

- Title: Numerical Linear Algebra in Optical Imaging
  Organization: Inter. Conference on Computational Mathematics
  Rio de Janeiro, Brazil. 1997

- Title: A Matrix Optimization Problem in Adaptive Optics
  Organization: Workshop on Numerical Methods in Optimization
  Curitiba, Brazil. 1997

- Title: Regularized Blind Deconvolution Using Recursive Filtering
  Organization: Workshop on Scientific Computing
  Hong Kong, China. 1997

- Title: Optimization Methods in Aero-Optics
  Organization: Workshop on Optimization and Numerical Methods
  Beijing, China. 1997

- Title: Large-Scale Computations in Optical Imaging
  Organization: SEAS-SIAM Annual Meeting
  Raleigh, NC. 1997
- Title: Inverse Filtering Methods in Blind Deconvolution
  Organization: Summer School on Multilevel Preconditioning in Scientific Computing
  Nijmegen, The Netherlands. 1997
- Title: Numerical Methods in Aero-Optics
  Organization: AFOSR Grantees Workshop
  Wright-Patterson Lab, OH. 1997
- Title: Blind Image Deconvolution by Inverse Filtering (Colloquium)
  Organization: UCLA Dept. Math.
  Los Angeles, CA. 1997
- Title: Iterative Blind Deconvolution using Variational Regularization (Plenary lecture)
  Organization: Annual Meeting of the Optical Society of America
  Long Beach, CA. 1997
- Title: Research in Aero-Optics (Three Lectures)
  Organization: Starfire Optical Range, Phillips Air Force Laboratory
  Albuquerque, New Mexico. 1998
- Title: Fast Algorithms for Phase Diversity Blind Deconvolution
  Organization: SPIE Conf. on Adaptive Optical System Technologies
  Kona, HI. 1998
- Title: High Resolution Imaging Through the Atmosphere (Colloquium)
  Organization: University of Kansas, Math Awareness Week Celebration
  Lawrence, KA. 1998
- Title: Low Rank Structured Operator Approximations (Invited)
  Organization: SIAM Annual Meeting
  Toronto, Canada. 1998
- Title: Computations in Areo-Optics Imaging
  Organization: AFOSR Grantees Workshop
  Wright-Patterson Lab, OH. 1998
- Title: High Resolution Imaging (Invited)
  Organization: SPIE Annual Meeting
  San Diego, CA. 1998

H Collaborative Research and Transitions at U.S. Air Force Laboratories

Recent activities for this work included visits to the Air Force Research Laboratory (AFRL) and Starfire Optica Range, Kirtland AFB, NM and to Wright Laboratory, Wright-Patterson AFB, OH. We are also in contact with DoD researchers at the Maui Space Surveillance Center (MSSC) in Hawaii. The primary contact at the AFRL is Dr. Brent Ellerbroek at the Starfire Optical Range, and we are collaborating on research involving closed-loop adaptive-optics systems. In two papers the authors have helped to develop a theory with possible applications for closed-loop adaptive control methods to adjust the shape of these mirrors in real-time [12, 41] . An paper on adaptive optics systems performance evaluation has been completed [39]. An abstract, "Leading Edge Methods in Optical Imaging", was prepared for the DOD publication, *Success Stories in High Performance Computing* in 1998, concerning our use of the Maui High Performance Computing Center's IBM SP2. Such collaboration is continuing, and a project involving optimal minimal variance estimators in adaptive-optics was just been completed in March 1999 [8].

We have also interacted with Dr. Julian Christou, Dr. David Tyler, and Dr. Donald Washburn at the AFRL on blind deconvolution and the airborne laser weapons program, and with researchers at Maui Space Surveillance Center (MSSC) concerning use of our methods for near real-time implementation in conjunction with their speckle imaging system. The MSSC has a direct line connection to the Maui SP2. Air Force Capt. Bruce Stribling is the primary contact at MSSC. Dr. Ellerbroek and Dr. Tyler at the AFRL have also supplied us with DoD satellite image data, sample phase screens, etc., for tests with our image post-processing work. Overall, we will be making available a robust, scalable software system that is portable across a variety of massively parallel computers and clusters of workstations.

Technology transfer of our work in relation to the projects at DoD Laboratories has included research on ground-based imaging of satellites, and related aero-optics activities. The PI has participated in three DoD workshops at the AFRL in New Mexico: (1) the *Aero-Optics and Image Reconstruction Workshop* concerning airborne laser weapons development, (2) the *Smart Sensors Workshop* concerning remote sensing for wide angle satellite surveillance, where the satellite systems will have on-board image processing capabilities when deployed, and (3) a second Workshop concerning recent trends in missile tracking for the airborne laser weapons program. Some of our work in these research projects concerns real-time adaptive filtering methods. Applications include closed-loop active noise (vibration) cancellation, with the potential for stabilizing firing pads for laser weapons.

In addition to the DoD applications described above, potential technology transfer of our research on these imaging projects to civilian technology include *astronomical and medical imaging, and remote sensing for commercial purposes.* In astronomical imaging there is technology transfer to the astronomical community at large. Medical imaging applications of the adaptive-optics work include fluorescence microscopy in three dimensions, and the use of adaptive-optics methods as an aid in deblurring images of the retina through the eyeball. Our image post-processing methods can also be applied to enhancing satellite collected images of the earth for agricultural,

law enforcement, and geophysical purposes.

During a visit by Plemmons to the DoD AFRL and Starfire Optical Range, February 25-27, 1998, three presentations were given: (1) Fast Algorithms for Phase-Diversity Based Imaging, (2) Adaptive Condition Estimation for Image Reconstructor Updating, and (3) Image Postprocessing by Constrained Inverse Filtering.

In addition, Plemmons and GRA Pauca, and others, visited the AFRL in New Mexico again on March 7-10, 1999. The purpose was to discuss the following topics in a workshop organized by Dr. Ellerbroek:

- Control algorithms for real-time adaptive optics (AO), performance optimization for a priori conditions, and adaptive methods for unknown conditions.
- Efficient numerical methods for AO system evaluation and optimization.
- Early tests on full amplitude and phase conjugation in aero-optics imaging from aircraft.
- Efficient algorithms for multiframe blind deconvolution and phase recovery for image restoration.

This latest visit was taken to this Air Force Research Laboratory *without AFOSR support*, which ended 12/31/98. Our purpose in participating in the workshop was to facilitate the completion of some joint papers involving Dr. Brent Ellerbroek at the Starfire Optical Range, and to meet with other researchers on optical imaging, as part of an informal meeting. We have a proposal pending to the ARO, and we envision that related applications of our work can be identified at, for example, the Army Research Laboratory at Adelphi, MD.

I Inventions or Patent Disclosures

None. The research sponsored by the AFOSR under this grant concerned the development of rigorous mathematical models, computational algorithms and high performance computer software implementations that will be made readily available to the DoD and the private sector.